denoting $\|A\| = \text{trace}(A^*A)$,

$$\|A^+B\| \leq \|X\| \text{ and } \|AA^+B - B\| = \min_X \|AX - B\|$$

for all solution X (see Ref. 9). The geometrical interpretation of Eq. (3) is as follows. The particular solution is the projection of X on the column space of A^* and the homogeneous solution its projection on the orthogonal complement of that space. In other words, they are perpendicular to each other. Therefore, we have $\|X\| = \|A^+B\| + \|(I - A^+A)Y\|$.

Application to System Identification

All matrices used hereafter are real and thus the conjugate transpose in the above section should be replaced by the transpose. Let M_a $(n \times n)$ and M $(n \times n)$ be analytical and improved mass matrices. The orthogonality condition of the normalized modes $\Phi^T M^\Phi = I$ can be rewritten as

$$\Phi^T W_1^{-1}(W_1 \Delta M W_1) W_1^{-1} \Phi = I - \Phi^T M_a \Phi \tag{4}$$

in which $\Delta M = M - M_a$ and $W_1 (n \times n)$ is any symmetric, nonsingular weighting matrix. Since the number of measured modes m is, in general, less than the number of degrees of freedom n, the measured modal matrix $\Phi(n \times m)$ is rectangular with full column rank m. Applying Eq. (3) twice to minimize $\|W_1 \Delta M W_1\|$ yields

$$W_1 \Delta M W_1 = U(I - \Phi^T M_a \Phi) U^T \tag{5}$$

where

$$U = (\Phi^T W_1^{-1})^+ = (W_1^{-1}\Phi)(\Phi^T W_1^{-2}\Phi)^{-1}$$

Equation (5) does not include the homogeneous solution because no other conditions must be satisfied. This is evident if we note that the improved mass matrix

$$M = M_a + W_1^{-1} U(I - \Phi^T M_a \Phi) U^T W_1^{-1} \tag{6}$$

is symmetric.

To identify the stiffness matrix, we start with the eigenequation

$$\Phi^T K = \Omega^2 \Phi^T M \tag{7}$$

and the constraint conditions

$$K^T = K \tag{8}$$

Define $\Delta K = K - K_a$ and $R = \Omega^2 \Phi^T M - \Phi^T K_a$. Since the analytical stiffness matrix K_a is symmetric, Eq. (8) implies that ΔK is symmetric. Postmultiplying Eq. (7) by a symmetric, nonsingular weighting matrix W_2 leads to

$$\Phi^T W_2^{-1}(W_2 \Delta K W_2) = R W_2 \tag{9}$$

The general solution of Eq. (9) is

$$W_2 \Delta K W_2 = QRW_2 + (I - QP)Y \tag{10}$$

where $P = \Phi^T W_2^{-1}$ and $Q = P^+$. Using the symmetry of ΔK, we get from Eq. (10)

$$Y^T(I - P^T Q^T) + W_2 R^T Q^T = (I - QP)Y + QRW_2 \tag{11}$$

Since $P(I - QP)Y = (P - P)Y = 0$, premultiplying Eq. (11) by P and rearranging yields

$$Y^T = QP(QRW_2 - W_2 R^T Q^T)(I - P^T Q^T)^+ \tag{12}$$

As noted earlier, the particular solution and the homogeneous solution in Eq. (10) are perpendicular; thus, minimizing $\|W_2 \Delta K W_2\|$ implies minimizing $\|(I - QP)Y\|$ or equivalently $\|Y\|$. Thus, the solution of Eq. (12) is given by

$$(I - QP)Y = (W_2 R^T Q^T - QRW_2) P^T Q^T \tag{13}$$

The improved stiffness matrix is obtained by substituting Eq. (13) into Eq. (10)

$$K = K_a + W_2^{-1} QR + R^T Q^T W_2^{-1} - W_2^{-1} QRW_2 P^T Q^T W_2^{-1} \tag{14}$$

In particular, when $W_1 = M_a^{-1/2}$ and $W_2 = M^{-1/2}$, Eqs. (6) and (14) reduce to

$$M = M_a + M_a \Phi(\Phi^T M_a \Phi)^{-1}(I - \Phi^T M_a \Phi)(\Phi^T M_a \Phi)^{-1} \Phi^T M_a \tag{15}$$

$$K = K_a + M\Phi(\Phi^T K_a \Phi + \Omega^2)\Phi^T M - (K_a \Phi\Phi^T M + M\Phi\Phi^T K_a) \tag{16}$$

which are the same as Berman's results.[1,2]

Acknowledgments

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The Fuel Property/Flame Radiation Relationship for Gas Turbine Combustors

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Introduction

TWO recently published papers[1,2] reported gas turbine combustor flame radiation data, and gave different correlations of these data with two fuel properties: weight percent hydrogen (H) and weight percent polycyclic

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aromaticity (PA). In the first paper, Clark[1] asserted that flame radiation is linearly related to hydrogen content and nonlinearly related to aromaticity. In the second paper, Naegeli et al.[2] argued that flame radiation is linearly related to both fuel properties. The purpose of this Note is to examine the recent Naegeli et al.[2] data to learn whether or not there is support for the nonlinearity argument.

Experimental Result

Figure 9 of Ref. 2 reports radiation fluxes for nine different fuels burning in the Phillips combustor. The hydrogen and polycyclic aromaticity percentages have been calculated in Table 1, and the corresponding radiation fluxes, taken from Fig. 9 of Ref. 2, are also shown.

Analysis

The data in Table 1 can be approximated by a single line of regression, if the following fitting parameter is used.

$$[\%H - m(\%PA)] \tag{1}$$

where m is an optimized constant. The value of m that best fits the Table 1 data is 0.055, and the corresponding line of regression is shown in Fig. 1. The fact that the correlation coefficient $r^2 = 0.99$ indicates that the quality of fit is quite good.

A second fitting parameter, which includes a nonlinear dependence on polycyclic aromaticity, is

$$[\%H - (\%PA)^n] \tag{2}$$

where n is an optimized constant. The line of regression that uses this second parameter to fit the data of Table 1 is shown in Fig. 2; the optimum value of n is 0.25. As with the first parameter, the correlation coefficient r^2 for the second parameter is 0.99, meaning that the fit is excellent.

In his paper, Clark[1] showed that a fitting parameter with a nonlinear dependence on polycyclic aromaticity provided an excellent correlation with Clark's data[1] and with earlier radiation flux data from Naegeli and Moses.[3] Clark concluded for a variety of operating conditions in a gas turbine combustor that the limits on the exponent n are

$$0.1 \leq n \leq 0.4 \tag{3}$$

The optimal value of the exponent n for the more recent data,[2] is within these limits as well.

To examine the implications of the linear and nonlinear fitting parameters, consider three fuels, all having a hydrogen content of 11.5%. The first fuel has a polycyclic aromaticity of 0.5%, the second 11.7%, and the third 50%. Table 2 shows the radiation fluxes which would be predicted by the linear

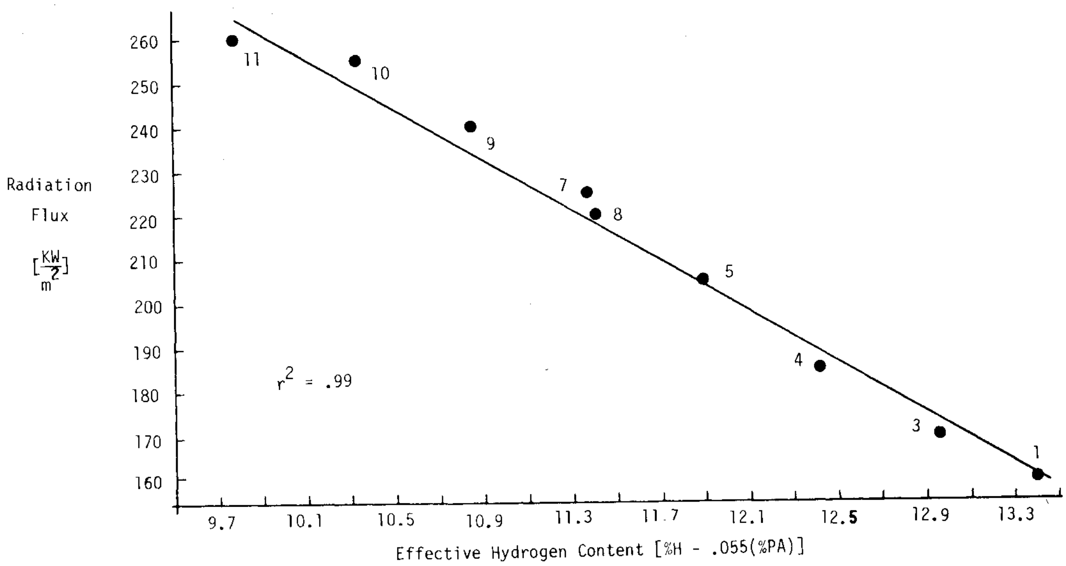

Fig. 1 Radiation flux correlation with linear fitting parameter.

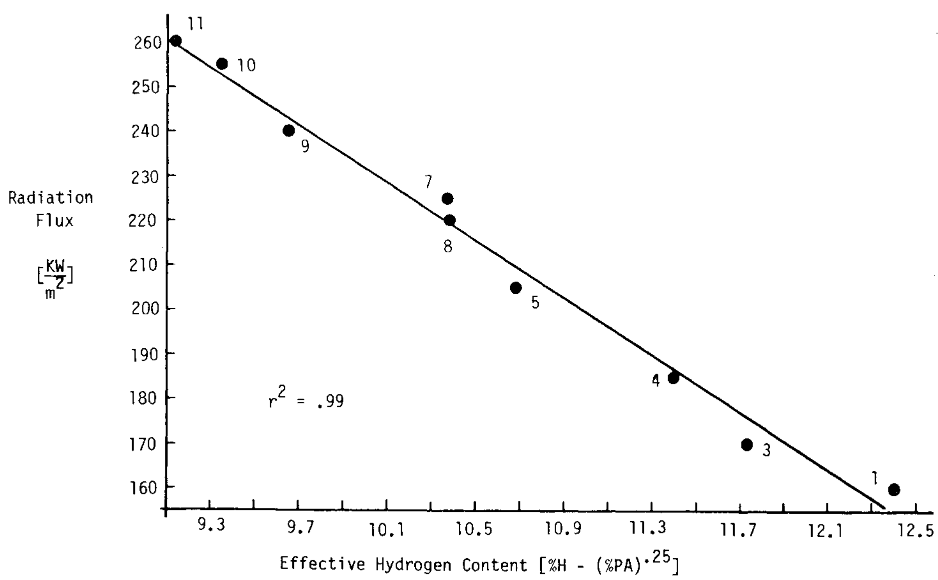

Fig. 2 Radiation flux correlation with nonlinear fitting parameter.

Table 1 Fuel and radiation data from Naegeli et al.[2]

Fuel No.	%H	%PA	Radiation, kW/m^2
1	13.5	1.39	160
3	13.5	9.75	170
4	12.5	1.48	185
5	12.5	10.96	205
7	12.5	20.52	225
8	11.5	1.57	220
9	11.5	11.72	240
10	11.5	21.28	255
11	11.5	31.21	260

Table 2 Predicted radiation fluxes for three fuels

Fuel	%H	%PA	Parameter 1 (linear) predicted flux	Parameter 2 (nonlinear)
A	11.5	0.5	220	205
B	11.5	11.7	240	240
C	11.5	50.0	305	265

and the nonlinear fitting parameters, for operating conditions matching those pertaining to the data in Table 1. Table 2 and Figs. 1 and 2 show that though the two fitting parameters predict nearly the same radiation fluxes for values of polycyclic aromaticity between 1.5% and 30%, they differ in their radiation flux predictions for fuels of very low or very high aromaticity. For both extremes, the nonlinear fitting parameter predicts radiation fluxes which are significantly lower than the radiation fluxes predicted by the linear fitting parameter.

To date, radiation flux data for petroleum-derived fuels of either very low or very high polycyclic aromatic content are virtually nonexistent in the literature. Clark[1] reported that JP-10, with a polycyclic aromaticity of only 0.04%, produced radiation fluxes which were considerably below the fluxes predicted by the linear fitting parameter but were in agreement with the nonlinear parameter predictions. Further evidence that gas turbine combustion radiation fluxes show a nonlinear dependence on polycyclic aromaticity may allow greater flexibility in the property specifications for combustor fuels. For example, instead of limiting fuel hydrogen to a minimum of 12% and polycyclic aromaticity to a maximum of 3%, a future fuel specification might recognize that a fuel with a hydrogen content of 11.5% and a polycyclic aromatic content of 0.5% can be burned without increasing the combustor radiation level.

It is important to state that this Note does not advocate either of the two correlating parameters for flame radiation; it simply points out that both parameters are equally excellent for predicting radiation fluxes. Furthermore, the mathematical complexity of the two parameters is identical, in that each parameter involves a single optimized constant. In other words, the fact that the nonlinear parameter yields correlation coefficients which are as good as those for the linear parameter is not due to a larger number of fitting constants in the nonlinear parameter.

A second remark is that this Note does not recommend the complete removal of aromatics from jet fuels as a means of reducing flame radiation fluxes. Some aromatics are necessary to prevent elastomer shrinkage in fuel systems; thus, it is unrealistic to speculate that future fuels will not contain aromatics. However, the possibility that future fuels might be refined to low levels of both hydrogen and aromaticity is a subject of continuing debate.[4] A significant reduction in flame radiation brought about by lowering the polycyclic aromaticity below 1% may be the incentive needed by fuel refiners to find low-cost methods for producing such fuels.

Conclusion

Radiation flux data from a recent paper by Naegeli et al.[2] have been shown to be excellently correlated with two distinct fitting parameters which involve fuel hydrogen and fuel aromaticity. One of the parameters is linearly dependent on polycyclic aromaticity while the second parameter has a nonlinear dependence. More radiation flux data from fuels whose polycyclic aromaticity is either very low or very high are needed to determine which fitting parameter is more correct.

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Errata: "A Low Mach Number Euler Formulation and Application to Time-Iterative LBI Schemes"

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DURING the preparation of the manuscript, the term $\rho^{-1} U \nabla \cdot \rho U$ was inadvertently omitted from the left-hand side of Eqs. (4) and (9) and should be included in these equations. This term was correctly included in the analysis and calculations reported in the paper.

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